Summer term 2026
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Linear and Integer Optimization

Exercise Sheet 3

Exercise 3.1: Apply the Fourier-Motzkin Elimination to decide whether the following systems of inequalities $A_i \leq b_i$ ($i \in \{1,2\}$) have a solution. If a solution exists then state a solution. Otherwise, give a y with $y \geq 0$, $y^t A_i = 0$ and $y^t b_i < 0$. You can use any elimination order of the variables.

(a) $A_1 := \begin{pmatrix} 2 & 1 & 3 \\ 1 & -1 & 0 \\ -4 & 1 & -3 \end{pmatrix}$, $b_1 := \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$.

(b) $A_2 := \begin{pmatrix} 1 & 5 & 0 \\ 2 & -3 & 1 \\ 0 & 5 & -1 \end{pmatrix}$, $b_2 := \begin{pmatrix} 8 \\ 2 \\ 10 \end{pmatrix}$. (3+3 points)

Exercise 3.2: Let $A \in \mathbb{R}^{m\times m}$. Show that exactly one of the following alternatives hold:

(a) There exists $x \in \mathbb{R}^n$ such that $Ax < 0$ (that is, every entry of Ax is strictly negative).

(b) There exists a vector $y \in \mathbb{R}^m$, such that $A^t y = 0$, $y \geq 0$ and $y \neq 0$. (4 Points)

Exercise 3.3:

(a) Consider the following linear program $\min\{c^\intercal x : Ax = b\}$. Show that it either does not have a solution, is unbounded, or that all feasible solutions are optimal. Does this statement hold if we additionally require $x \geq 0$?

(b) Let (P) be a linear program of the form $\max\{c^t x \mid Ax \leq b\}$. Show that the dual of the dual of (P) is equivalent to (P). (3+3 points)

Exercise 3.4: For $A \in \mathbb{R}^{m\times n}$, $c \in \mathbb{R}^n$ and $b = (b_1, \ldots, b_m) \in \mathbb{R}^m$ let $x^* \in \mathbb{R}^n$ be an optimum solution of the LP $\max\{c^t x \mid Ax \leq b\}$. Moreover, let $\tilde{b} = (\tilde{b}_1, \ldots, \tilde{b}_m) \in \mathbb{R}^m$, and let $\tilde{x} \in \mathbb{R}^n$ be a vector with $A\tilde{x} \leq \tilde{b}$. Assume further that $a_i^t \tilde{x} < \tilde{b}_i$ implies $a_i^t x^* < b_i$ for all $i \in \{1, \ldots, m\}$ (where a_i^t is the i-th row of A). Prove that $\tilde{x}$ is an optimum solution of the LP $\max\{c^t x \mid Ax \leq \tilde{b}\}$. (4 points)

Submission deadline: Tuesday, May 5, 2026, 16:00, via eCampus (in groups of at most 3 students).